# Mathematical modeling: characterization of a proposal for the teaching and learning of multiplication and division

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#### **Abstract**

This is a work which has the purpose to provide input about a proposal for teaching Mathematics in particular multiplication and division, taking into consideration Mathematical modeling starting from a characterization and analysis from the educational reality. Currently, educational models focus on the students building mathematical knowledge through interaction with their environment, where the teacher intervenes as a guide who intentionally plans and teaches mathematical knowledge. Contrary, the results of our country in international assessments such as PISA 2012 (INEE, 2013) suggest that students demonstrate low levels of performance in learning Mathematics. So, it is necessary to set teaching strategies that involve solving problems and that at the same time they work as learning vehicles and not as an end in themselves. This study emphasizes that a child learns by observing, manipulating, validating and building. Some studies have shown that students must learn in relation to what they live day by day. It also shows strong aspirations to propose a possible alternative in the teaching of mathematics: mathematical modeling.

## Mathematical modeling, mathematical problems, teaching and learning of multiplication and division

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### Introduction

The present world forces the construction of different visions on reality and the different forms of problem solving using reasoning as a fundamental tool (SEP, 2011). Representing a solution involves establishing a set of symbolisms and correlations through mathematical language.

Knowledge of rules, algorithms, and definitions formulas will only considered important insofar as students can use them in a flexible and reflective way to solve problems that are significant and relevant to them. In other words, it does not make sense to have a closed and linear mathematical language if at the moment of confronting a problematic one does not have the capacity to flexibilizar the language to give solution to situations that are exigible in its daily social development.

That is why it is necessary to carry a process of learning of mathematics in which it starts from the informal knowledge to enter into the formal mathematical language and its ramifications to arrive at the conventional one; In other words, to use what is learned in everyday life. The fundamental intellectual activity in these processes must have a basis in reasoning and logic and not only in memorization (SEP, 2011).

Currently the educational models focus on the students to construct mathematical knowledge through interaction with their environment, where the teacher is inserted as a guide, as the subject that intentionally plans and didactifica mathematical knowledge. However, our country's results in international tests such as PISA 2012 (INEE, 2013) indicate that students demonstrate low levels of performance in mathematics compared to most of the other countries evaluated.

In this way it is possible and pertinent to propose a teaching and learning of mathematics that involves different ways of solving a problem or the use of different strategies when facing problems of multiplication or division.

# The teaching of multiplication and division algorithms in school reality in primary education

We are living in a world in which the impact of mathematics on social culture is very evident. Mathematical knowledge has been facing the adaptation of use of algorithms and operations for the various modern and current tools of the 21st century; For example, machines (some more sophisticated than others), interpretation on the stock exchange, (VAT, buying and selling currency, taxes, etc.), understanding the time, building a house, algorithms that make up the organization Digital on the internet, etc.

In this way, the influence that mathematics has for human development is clearly visualized. On the one hand, it is a science that serves to generate knowledge and perception, and; Complementing, is a system of instruments, products and processes that favor the exercise of a wide range of techniques and practices (Hernández Pina & Soriano Ayala, 1999).

One of the purposes of mathematics in primary school is to create a positive attitude towards students, and one of the means to achieve this is to help children experience intellectual pleasure through them. However, today and contrary to the above, children develop from primary education, a barrier based on the fear of mathematics (Gómez-Chacón, 2000). Within this barrier are introduced several problems such as the mechanical memorization of algorithms for Solving problems of multiplication and division at best.

This reality is what makes possible the need to achieve a series of actions to characterize the processes of knowledge construction in the mathematical field, with emphasis on the basic algorithms, in this case multiplication and division.

The problem is seen, in the rote use of a single form in the resolution of operations that involve division, multiplication or both. That is why one of the most important mathematical rules is broken: The most important thing is not to arrive at a solution, but the processes that the students follow when trying to find it. (Hernández Pina & Soriano Ayala, 1999).

## Methodological perspective

Qualitative research has its origins in anthropology, seeks a holistic understanding, and puts the emphasis on depth (Bizquerra, 1989), based on it, is assumed as a methodological feature relevant to the present research work. The techniques of data collection that were used were the in-depth interview.

The interview consisted of open-ended questions where the teacher answered what he thought would be the procedure to learn and / or teach problematic situations regarding multiplications and divisions, explaining further how the result would be achieved. This served to gather information about their strategies in the development of basic operations and the understanding of problems.

The research that is presented was developed in a primary school located in the community of El Tepetate, Loreto, Zacatecas; In the framework of the days of professional practice for teachers in initial training of the 5th Semester during the school year 2015-2016 in the Degree in Primary Education. The subject of study for this stage of work is the teacher in charge of the groups of 3 ° and 4 °.

It is to be understood that in the second cycle of primary education one begins to know in depth the use of algorithms of multiplication and division.

### **Problem formulation**

### **Central question**

How to improve current skills, strategies, techniques or resources for teaching and learning multiplication and division in the third and fourth grades of primary education?

- Complementary questions ¿How to introduce the reflection and analysis in the different problems that entail the algorithm of the division and multiplication?
- What strategies lead to reflective learning through collaborative and / or individual work?
- How to get students to relate the two algorithms-multiplication and division-as something similar and in turn opposed?

### **Objetives**

Identify and recognize the problems of the students through the diagnostic evaluation and the investigative and theoretical background to enable an improvement plan based on reflection, analysis and understanding.

Identify learning strategies where children understand the use of division and multiplication in everyday life.

Modeling the learning and teaching of mathematics. Current research perspectives

A child learns by observing, manipulating and constructing, from the behavioral view of teaching and learning mathematics (traditional teaching) is irrelevant.

Different research has shown that students should learn in relation to what they live day by day. Here we enter a situation that gives way to a possible alternative of response to the change of the teaching of mathematics: mathematical modeling.

Modeling in the mathematical education environment refers to the process involving the representation of reality by means of a model. There is mathematical often a distinction between approaches to modeling, on the one hand, we have applications and modeling for learning mathematics and, on the other hand, learning mathematics to develop skills in the construction of mathematical models. The first slope considers the use of modeling activities as a vehicle for the construction of mathematical concepts. And the second slope involves the application of mathematics to construct mathematical models (Guerrero Ortiz & Mena Lorca, 2015).

Modeling in the mathematical education environment refers to the process involving the representation of reality by means of a mathematical model (Guerrero Ortíz & Mena Lorca, 2015). But it should also be borne in mind that the use of modeling in school is shown in different ways according to the points of view from which the didactics are viewed (Trigueros Gaisman, 2009). The same author, quoting Freudenthal (1968), argues that, if mathematics really desires value, they must be attached to reality and relevant.

(Trigueros Gaisman, 2009) Emphasizes what Camarena 1999 and 2000 says: the modeling process is conceived as a whole and not as something partial, whose objective is the development of approaches to the way in which the Applied mathematics and not the development of concepts. The researches of Trigueros Gaisman emphasized that the anthropological theory of didactic (TAD) proposes that for each mathematical activity a modeling of the same one can be done.

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Likewise (Sierra Delgado, Bosch Casabó, & Gascón Pérez, 2013) establish that the TAD postulates that all mathematical activity can be interpreted as an activity of study and production of praxeologies with the aim of answering certain problematic issues. This activity requires that the student (whether a student, a teacher or an investigator) has access to or, if appropriate, constructs certain appropriate mathematical techniques and can use, when required, a mathematical discourse to interpret, make sense and evolve Mathematical practice.

(Trigueros Gaisman, 2009), establishes that one of the objectives of this work is that students need to be able to solve a problem or situation in different ways.

In this case, the central objective of this research is emphasized: different ways of solving a multiplication or division operation. That although it is something very limitante to say it of that form would be necessary to include all the process that entails. The development of research involves all the steps that must be followed in mathematical modeling as mentioned (Herrera Villamizar, Montenegro Velandia, & Poveda Jaimes, 2012) cited by (Trigueros Gaisman, 2009):

The definition of the model: identify variables and constants of the problem, including the identification of what varies and what remains constant.

Formulation of the algorithm: establish relations between variables and constants through the concepts involved in the problem.

Develop the program: validate the "mathematical relationship" that models the problem, which is done by returning and verifying that it involves all the data, variables and concepts of the problem.

Complementing the previous arguments and relating them to the main theme (Calvo Ballestero, 2008), he mentions that the methodology used in teaching problem solving in mathematics is a key element for the satisfactory achievement of the contents in this subject. Therefore, by relating modeling as a teaching strategy to solve multiplication and division problems, it can have an effect on favorable learning outcomes. Unfortunately in the teaching of mathematics, analytical and reflexive thinking has been neglected, which replaced memory has been by generated mainly mechanization the repetition of exercises (Calvo Ballestero, 2008).

When making a recapitulation, in the midst the continuous transformations in our country, it is not enough to have specific knowledge and exercise its transmission. It is essential to build experiences and learning opportunities that enable new knowledge and skills in their application and socialization, both in the teaching activity: in analysis and progressive the reflection, innovation of professional practice, as well as in the impact that would be envisaged in better learning and Greater of content mathematical understanding. In view of the above, mathematical modeling is assumed as a relevant methodology for teaching and learning specifically mathematics, solving multiplication and division problems in the third and fourth grades of primary education (Salett Bienbengut & Hein, 2010).

# Theoretical perspectives in teaching and learning multiplication and división

## The importance of logic and reasoning in learning basic operations

The writings of Jean Piaget in his book "The conception of number in children" mentions that logic is the most solid criterion for defining numerical comprehension of children (Nunes & Bryant, 1997).

From here the bases of why pupils should keep in mind the order of numbers are highlighted. There needs to be a transitivity to understand the nature of the number.

The curious thing about mathematical reasoning is that it involves mixing a general logic. Anyone should understand that 1 + 1 = 2 because a comparison is made in the order of numbers and is logically demonstrated. However, the development of mathematical logic (which in turn also develops reasoning) implies that it is related to its context. If the students are aware that daily they are involved in the use of the basic operations, the teacher needs to take the daily problems to the classroom to solve them in a reasonable way.

An intrinsic process of the student should be carried out where he asks questions, analyzes data, makes decisions and values his own knowledge. You need to develop your mental agility little by little to make small mental accounts involving addition and subtraction, to subsequently reach a complex level and develop operations of division and multiplication.

### **Development of mental calculation**

Most of the calculation that is done daily outside of the school is mental. Many times the answer does not have to be exact, it is enough with an approximation. This type of calculation has certain characteristics:

- Can be done quickly
- It relies on a limited set of numerical facts
- Requires certain skills (counts, relocations, compositions, decompositions, etc.)

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Certainly this is not a goal for school, sophisticated methods of mental calculation are inappropriate for children's minds, but that does not mean that from the outset the foundations cannot be laid to achieve at the end of schooling a dexterity, efficacy and Reasonable speed for the most common calculation situations (Gómez Alfonso, 2007).

Thus, from an early age in the introduction to mathematics, concentration, habit, attention, and interest should be involved to achieve favorable results in this type of calculation. An example of this is the operation of permanent activities of addition and subtraction operations beginning with numbers of a figure and increasing the difficulty according to the speed and agility of the student.

## One of the bases of multiplication: The multiplication tables

A good mental calculus skill is not possible if good support points are not available. One of them is known as the tables. Thanks to them it is possible to calculate without worrying about the size of the numbers as soon as the methods that allow to reduce the manipulation of the numerical symbols are dominated to those that appear in them.

(Gómez Alfonso, 2007) It points out two ramifications as to how to teach the tables. One based on memorization or blind and the other based on personal strategies. In order to decide on the most appropriate line of action, it should be borne in mind that one approach leads to the other. Although this does not occur in both directions. The use of strategies may end up in memorization of results, but memorizing results not only does not lead to the design of strategies, but obstructs them.

Whatever the case may be, it should be pointed out that multiplication tables are a permanent exercise of mental calculation and a primary basis for initiating the basic operations of multiplication and division. Operations that to be able to solve them properly require a strong foundation in addition and subtraction, mental calculation, counting and reasoning. This is where all operations are articulated in a balanced way where basically multiplication is the sum of large numbers and division is the subtraction in equal parts of different types of numbers.

### Introduction to multiplication and division

The introduction to multiplication and division does not mean knowledge of multiplication tables, or the mechanics of these operations; it is intended only to arrive intuitively at the concept of multiplication as a "special sum", that is, as sum of equal sums, and to the concept of division as a distribution of equal parts or as successive subtasks (Cascallana, 1993).

Product learning and division is the beginning of the study of a new structure. However, as already mentioned in the previous section, starting to work on multiplication and division requires that the student has a level of use and mastery of numbers, which knows its symbolization, all to a more complete degree than in Case of addition and subtraction.

Multiplying is iterating an amount, at its most intuitive level. To divide is to distribute a quantity in equal parts (Castro, Rico, & Castro, 2007). The dividend is the amount to be distributed and the divisor is the number of shares.

In the beginning of these two basic operations (Nunes & Bryant, 1997) they mention that, the teaching needs to concentrate on creating situations that serve as a bridge between the understanding of children and the new strategies that employ functional or scalar factors expressed in a multiplicative way. On the other hand (Castro, Rico, & Castro, 2007) they refer that we must leave two years or intermediate courses between the teaching of addition-subtraction and multiplication-division to strengthen the first two operations.

# New methodology in the restructuring of learning and teaching mathematics: mathematical modeling

For some years now, restructurings in the curriculum and teaching strategies of mathematics have been processed, according to (Biembengut & Hein), among other aims, is to increase interest in its application in everyday situations, in addition Which visualize that learning is not just adding knowledge, rather it is assumed as a "process of growth", "Knowing is a process and not a product."

In mathematical modeling, the theme is unique and from it the programmatic content is extracted. It can be used even during the whole school period. As long as you have enough content to develop the program and do not exhaust the motivation of the students. The suggestion is that the method has the following sequence: justification of the process, choice of topic, development of program content, analogous examples - concept setting and evaluation and validation of results (Biembengut & Hein).

### **Process justification**

It begins with a critical analysis on the conventional teaching of mathematics and shows the possibility of presenting the mathematical content from real situations, thus giving a practical sense.

For this the teacher exemplifies exposing a known mathematical model and directs his exposition in a way that clarifies what are the mathematical concepts and operations that become necessary for the understanding of the proposed situation. Above all, it is necessary to find effective ways to motivate students so that they voluntarily decide to actively develop learning.

### **Choice of topic**

As the students are suggesting topics, a list is made on the board for a later election. The teacher can also use to interleave some topics (as a suggestion), mainly those that are already known in relation to the breadth of its contents. The choice of the subject by the students will make them feel participants in the process.

The performance of the teacher should be primarily focused on the use of strategies that facilitate the choice of a broad, motivating subject and on which, in a way, it is easy to obtain data or information. For example, construction of houses, corn plantation, travel in a trip, etc.

### **Development of pragmatic content**

We will say for example that the students chose the theme of building houses, the teacher should look for strategies to develop the content correctly. The teacher can start with a detonating question: What is necessary to build a house? Then you can start a topic of geometry based on the construction of a house and developing subtopics of basic operations, perimeter, area, etc.

### **Evaluation**

A formative, progressive and permanent evaluation is applied in which children are free to explain and to base their answer, then it is there that the knowledge is constructed because the other children will be able to validate that their work is correct confirming or mathematically rejecting the arguments and the Teacher who will be a mediator will have them prove using the learning they have developed.

## First approaches to relevant categories for mathematical modeling

## Mathematical knowledge. Conceptualization on multiplication and division

The domain of mathematical content is a fundamental element in the learning and teaching of mathematics. With what knowledge of structure, subjects and mathematical practice can a mathematical problem be solved? Thus, within the interview that was applied to the professor subject of the investigation were verified some of the concepts that has on the multiplication and division.

- 1 What is multiplication?
- 2 It is to duplicate one or many times an amount. It can also be through the same

3 sum.

- 4 What is division?
- 5 Disbanding an amount in equal parts or sharing an amount.

One of the mathematical competencies of teachers is to have the ability to understand conceptually; So that in later occasions it can represent and relate different parts of the mathematical content and use it in problem solving (Chamorro, 2006). As we can observe in the questions that were asked to the teacher, he answers correctly, indicating the knowledge he possesses about the mathematical content.

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### Troubleshooting

Solving mathematical problems depends on two fundamental aspects: the operation as theory / practice at the same time and the implication of solving applied problems. The following sections will analyze in depth the development of how an operation is solved depending on its location: solitary or in a situation.

### **Individual operations**

Individual operations can be taken initially as a teaching of the theoretical to the practical without a present situation, it is important not to assume it as isolated execution exercises of practical. mathematical and probably understanding comprehension, rather knowledge of structure and Mathematical practice that at a certain moment is pertinent to the knowledge of the mathematical content. Taking the code P1E1 we placed the procedure that followed the teacher to solve some solitary operations on the multiplication and division:

- 6 How much is 28x4?
- 7 We initially placed the operation in order on the board to solve it. After
- 8 multiply 4x8 and put the number of units below the line. The amount
- 9 of the hundreds put it above 2, then multiply 4x2 and result
- 10 add the number that we have above the 2, then lower the number below
- 11 the line and we have 112.

In the evidence, the teacher uses the technique of vertical multiplication, something that is commonly done in schools Why not teach multiplication by the method of the ancient Maya or by "gelosia" as mentioned (Sierra Delgado, Bosch Casabó, Gascón Pérez, 2013), who determines another method and uses the sum table.

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However, whatever the method to be used in multiplication, to improve the appearance of this, it is proposed to use mathematical modeling.

As already stated, mathematical modeling improves teaching techniques through context or real situations. In this process the context is part of the learning process (Trigueros Gaisman, 2009).

With regard to the division is the following part of the interview:

- 17 How much is 65/5?
- 18 We put the corresponding numbers in the "little house" and divide 6/5, making the
- 19 Question: How many times does 5 fit in 6? As it is one, we put the one in the
- 20 ratio and multiply 5x1, then 5 we put it under 6 and subtract.
- 21 One is left and we get down to 5, then we have the number 15, now we repeat: How many
- 22 times it fits the 5 in the 15? The answer is 3 and we put it in the quotient, we multiply
- 23 and we subtract and as the subtraction is 0 the operation is finished.

The procedure of this basic operation is common to see in elementary school. In this respect we cannot reiterate that in order to comply and do this technique correctly we must first be clear about the iterated sum and multiplication tables, as well as the notion of apportioning. Usually this operation can be used by defragmenting the dividend and making smaller distributions. However. procedural use as planned is meant to be memorized. Here comes the ability of each student to modify the procedure according to their knowledge and mathematical skills.

Emphasizing that the teacher must be aware of how students do and validate through the socialization of mathematical knowledge in play.

## **Problems applied**

- 32 Ulysses goes to the store and buys 9 sabritas at a price of \$ 6.00 and 15 sodas with a
- 33 price of \$ 12.00 How much did you spend at the Ulysses store?
- 34 This problem initially involves an iterated sum of 9 times 6 and 15 times 12 o
- 35 a multiplication obtaining 54 in the first account and 180 in the second. So
- 36 if you ask the total price to pay we must make a final sum of 54 + 180 for
- 37 get the result they ask us leaving a total of 234
- 38 Anahí has a bag of sweets, the moment she decides to count them she discovers that she has
- 39 40 candies. If you decide to distribute them among your 8 friends. How many sweets will you touch
- 40 every friend of Anahí?
- 41 It is necessary to take the data that they give us and to analyze that if it wants to divide in leaving we must
- 42 make a division. Then take two facts: How much are we going to split? Y
- 43 Between how many? Reading the problem again we are going to divide 40 sweets between 8
- 44 friends, then we execute the division having a result of 5.

We can see that the teacher concentrates on the location of data and the analysis of what he asks. We must be clear that, at some point, the information contained in some problem approach may exceed the student's cognitive resources or away from the schemes that were relevant in simpler cases (García Alcalá, Vázquez Maldonado, & Zarzosa Escobedo, 2013). In this case students can add up the amounts that they have without paying attention to what is asked.

The improvement of these types of problems is that they have relevance to children's lives. In this case the mathematical modeling comes into play and thus through this, the students generate a logical reasoning for which it facilitates the understanding of the problem. In the case of Ulysses' situation, the teacher briefly introduces him because he already knows what he has to do. A student reading it and seeing that a real and everyday situation is present can solve it easily or with less help.

The student can solve it initially with the knowledge that he has no matter how long or short the procedure. What is valid is the result and the process that followed. Arguments such as (Cascallana, 1993) in his research mention that students need the time needed to solve their problems and adequate patience to take the error as new knowledge.

#### Conclusions

The teaching of mathematics is a subject that is taking a lot of relevance for the evaluation results in the primary schools of the country. Students see math as a headache and many of them contain a barrier to learning about this subject. This can be, among other factors, a traditional teaching of mathematics in which improbable data, unrealistic situations and the emphasis on mechanical memorization are induced.

The theoretical discussion, which contemplates some researches such as those presented in this work, show that mathematical modeling, based on real situations, on topics of interest in children and on various procedures for improving the themes of mathematics. We must start from real situations to demonstrate a topic, for example, the case of basic operations improvement so that students know that mathematics serves daily life.

Likewise, the justification and explanation in each procedure is fundamental to validate the constructed mathematical knowledge, besides serving as feedback. In the case of multiplication and division, since the operations are contrary and at the same time related, justification serves in two aspects: to justify the result and to feed back the opposite operation.

In this way, an initial response is formed, as a characterization, on the improvement of multiplication / division teaching through mathematical modeling, which responds to different issues such as problem justification, collaborative work as a means to socialize strategies Of procedures and the use of different real situations that involve all the basic operations through themes, for example, the construction of a house.

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